Reprinted from JOURNAL OF APPLIED METEOROLOGY, Vol. 5, No. 1, February 1966, pp. 43–48
American Meteorological Society
Printed in U. S. A.

A Comparison of Tower and Pibal Wind Measurements

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(Manuscript received 22 July 1965, in revised form 20 November 1965)

ABSTRACT

An empirical study of the time and space variability of the wind as measured by a tower-mounted wind instrument and triple-theodolite pilot balloon observations at Green River, Utah, during the months of June and July 1964, is presented.

Simultaneous wind measurements at 500 ft above the ground are compared. Mean differences in wind direction was 15.0 deg with a sample standard deviation of 12.2 deg. The mean speed difference was 6.2 ft sec^{-1} with a sample standard deviation of 5.5 ft sec^{-1}.

1. Introduction

Wind measuring instruments have been mounted on towers to varying altitudes up to approximately 1500 ft in order to obtain a wind profile. Another system used to obtain winds in this layer is the pilot balloon observation technique where one or more theodolites track a balloon. Since the first system is in essence an Eulerian type measuring system and the balloon system is a semi-Lagrangian measurement, the question arises as to the compatibility of the wind profiles generated by these systems. That is, if both systems are measuring the wind perfectly, then the wind should be exactly the same at any given point along the profile.

Experimentally it has been found that large differences are present in the profiles generated by the two systems involved. It is realized that many factors can influence the resultant wind, among which are space and time variability, terrain effects, systems error, balloon aerodynamics, tower shadow effect, and reduction processes. It is beyond the scope of this paper to attempt to determine the extent of influence of any one of these factors on the wind profile. The purpose of this study is to show the "apparent" wind differences at a given level of the profile without attempting to determine the cause. The level selected for this study was the 500-ft level utilizing a three-cup anemometer and a three-theodolite wind measuring system.

2. Data collection and reduction

The 500-ft tower is located approximately 400 ft from a missile launch pad at Green River, Utah. Only the data from the anemometer at the 500-ft level were utilized in this study since the problem concerns the differences in the measured wind at the top of the tower (an Eulerian system) and the pibal measured wind (approximate Lagrangian system) at the same level. The data from the anemometer were transmitted electronically to a digital computer at a sampling rate of one point per second and averaged over a 10-sec period centered around the time at which the pilot balloon reached a height of 500 ft to provide a mean wind for comparison. The thickness of the layer traversed in a 10-sec period varies with ascent rate but is generally 140 to 180 ft.

The pibal system consisted of three semiautomatic theodolites placed as an equilateral triangle with sides of approximately 3345 ft. In this system the balloon was visually tracked by an observer, and the azimuth and elevation angles were electronically transmitted on a real time basis (Duncan, 1963) to a digital computer at White Sands Missile Range, providing a wind value each second. These wind data were also averaged over a 10-sec period centered around the time the balloon passed through the 500-ft level. The pibal winds were calculated utilizing Duncan's mathematical derivation which is presented in the Appendix.

3. Discussion

The tower wind is a measurement at a fixed point. This wind is averaged over a 10-sec interval. The pibal wind is a mean wind also over a 10-sec interval (a layer approximately 170 ft thick) centered at the 500-ft height which involves horizontal as well as vertical space. It has been stated (Panofsky and Lumley, 1964) that Lagrangian and Eulerian wind variances are theoretically equal in stationary, homogeneous, incompressible turbulence. A comparison between tower and airplane wind measurements at heights of 295 ft and 394 ft by Lappe *et al.* (1959) showed that the theory appears to be valid for horizontal wind fluctuations.

The surface roughness at Green River (Armendariz, 1965) precludes homogeneity of the atmosphere in the lower layer. It is felt that considerable mechanical turbulence exists with a dominance of eddy sizes of a

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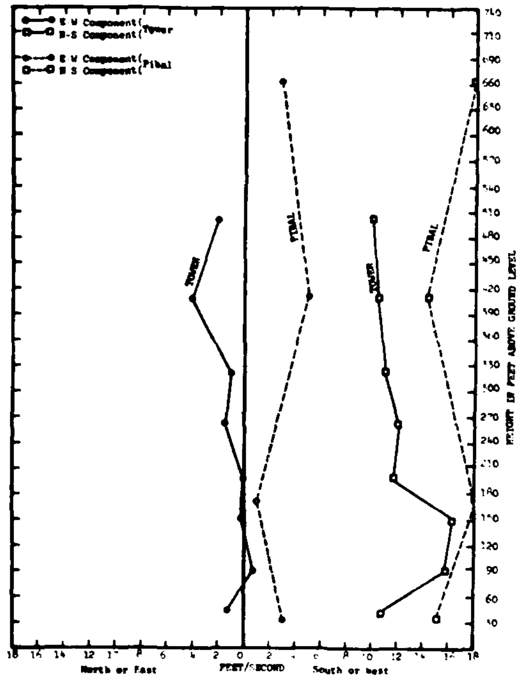

FIG. 1. A typical low-level wind profile comparison between pibal and tower wind measurements at Green River, Utah.

scale less than the distance between the tower and the balloon.

In a study by Moses and Daubek (1961) it was concluded that towers distorted the wind flow thereby producing errors in wind velocity measurements obtained by anemometers mounted on them. In some cases they found that the tower-mounted anemometer gave wind speed readings appreciably higher than a reference anemometer, and in other cases for given wind directions, there was a substantial reduction in the wind speed recorded from the tower as compared to the reference anemometer. This tower wind-shadow effect reduced the wind measurements nearly one half in light winds and nearly 25 per cent for speeds between 10–15 mph. Preliminary results in the wind tunnel at Colorado State University (Cermak, 1964) showed a reduction of 10–15 per cent for the same speed range. There is also some evidence that the ordinary pilot balloon does follow the actual wind flow (MacCready *et al.*, 1964; Scoggins, 1965; Killen, 1960) and therefore is not a true Lagrangian system.

4. Results

The mean direction difference (Table 1) for the 71 cases was 15.0 deg, with a sample standard deviation of 12.1 deg. The greatest mean difference recorded was on 12 June when the mean difference was 25.3 deg and the least was on 14 June, a mean of 7.1 deg.

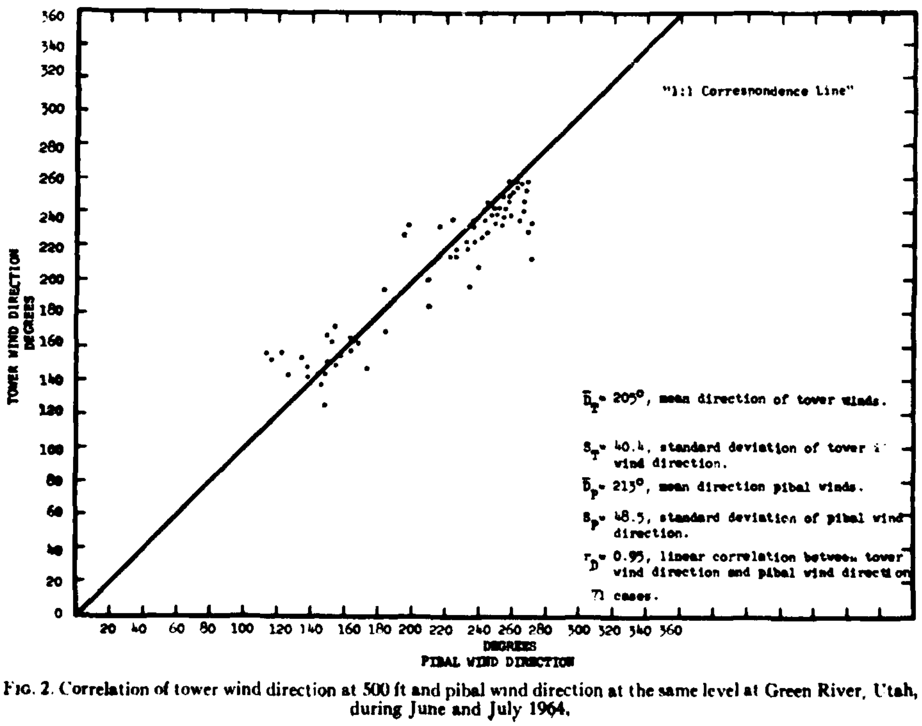

FIG. 2. Correlation of tower wind direction at 500 ft and pibal wind direction at the same level at Green River, Utah, during June and July 1964.

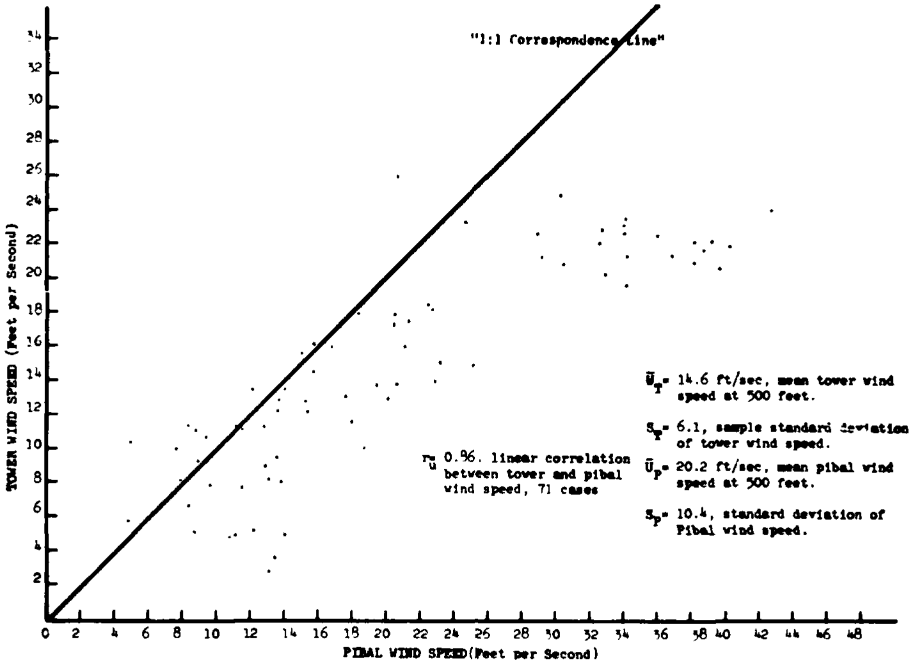

FIG. 3. Correlation of tower wind speed at 500 ft to pibal wind speed at the same level at Green River, Utah, during June and July 1964.

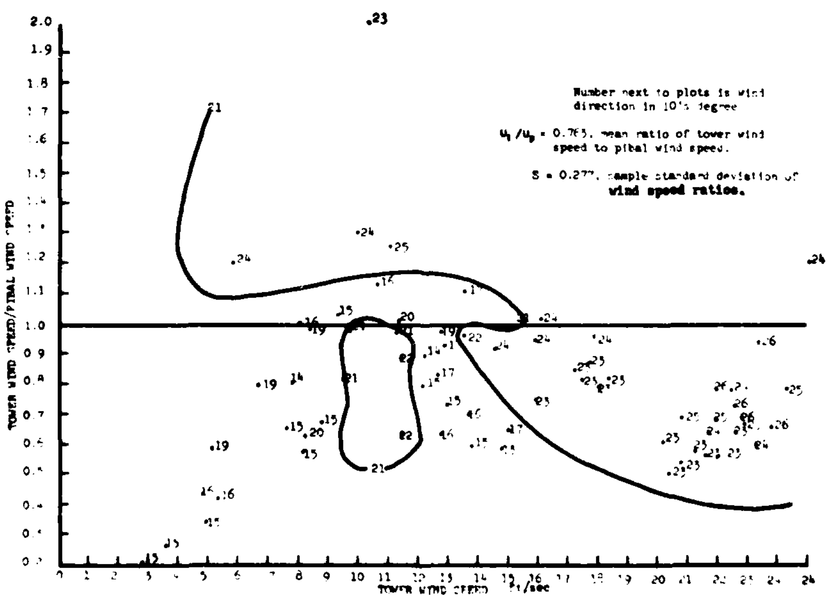

FIG. 4. Relation between the ratio of tower wind speed to pibal wind speed and tower speed at 500 ft at Green River, Utah.

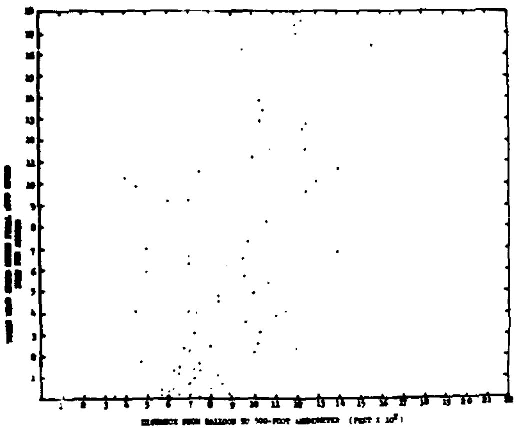

FIG. 5. Relation between the difference in wind speed of tower and pibal at 500 ft to distance from balloon to 500-ft anemometer. Green River, Utah. 71 cases.

The mean wind speed difference was 6.2 ft sec^{-1}, with a sample standard deviation of 5.5 ft sec^{-1}. The greatest mean speed difference occurred on 17 June with a mean speed difference of 14.1, whereas on 12 June the mean was only 2.1 ft sec^{-1}.

The linear correlation of the speed and direction between the tower and the pibal wind was $r_u = 0.86$ and $r_D = 0.95$, respectively.

It is interesting to note for the case illustrated in Fig. 1 that the direction of the east-west component for the pibal is generally opposite to that of the tower, and the speed of the north-south component is less for the tower than for the pibal. These differences are indicated in Figs. 2 and 3.

TABLE 1. Pibal and tower wind comparison at 500 ft, Green River, Utah. Direction difference is in deg; speed difference in ft sec^{-1}.

Dates	Observations	Wind direction $D_T - D_P$	Wind direction $S_{(D_T-D_P)}$	Wind speed $U_T - U_P$	Wind speed $S_{(U_T-U_P)}$
10 June 1964	10	9.2	5.7	11.1	5.0
12 June 1964	12	25.3	16.4	2.1	1.9
14 June 1964	12	7.1	6.9	3.0	2.6
15 June 1964	11	17.9	12.4	6.7	3.0
16 June 1964	9	19.6	11.6	2.4	2.7
17 June 1965	12	11.0	5.3	14.1	4.9
8 July 1964	5	18.4	4.6	2.4	2.2
Total	71				

$\overline{(D_T-D_P)}_{71} = 15.0$ $\quad S_{U_T} = 6.1$ ft sec^{-1}
$S_{(D_T-D_P)71} = 12.2$ $\quad S_{U_P} = 10.4$ ft sec^{-1}
$\overline{(U_T-U_P)}_{71} = 6.2$ $\quad S_{D_T} = 40.4°$
$S_{(U_T-U_P)71} = 5.5$ $\quad S_{D_P} = 48.5°$
$\bar{U}_T = 14.6$ ft sec^{-1} $\quad r_u = +0.86$
$\bar{U}_P = 20.2$ ft sec^{-1} $\quad r_D = +0.95$
$\bar{D}_T = 205°$ $\quad S$ = sample standard deviation
$\bar{D}_P = 213°$ $\quad D_T$ = wind direction of tower
D_P = wind direction of pibal
U = wind speed
r = correlation function

Fig. 2 shows the majority of the wind directions plot to the right of the 1:1 correspondence line. In Fig. 3 most of the wind speeds are also to the right of the 1:1 correspondence line, indicating that the tower is generally reporting lower wind speeds.

Fig. 4 is a plot of the tower wind speed as abscissa and the ratio of the tower wind to pibal wind as ordinate. In 59 of the 71 cases the ratio was less than one, indicating a lower value for the reported tower wind than for the pibal. The ratio is significantly less than unity for southeasterly wind speeds of less than 15 ft sec^{-1}, and for southwesterly winds with speeds greater than 15 ft sec^{-1}.

Fig. 5 is a scatter diagram of the differences between the tower and the pibal wind speeds as a function of the balloon distance relative to the tower at the time the balloon reached the 500-ft level. As would be expected, an increased wind speed difference with distance is indicated.

5. Conclusion

Differences exist in wind observations obtained with tower-mounted anemometers and with pilot balloons at the 500-ft level. Mean direction difference was 15.0 deg and the mean speed difference was 6.2 ft sec^{-1}. Good agreement between the wind directions from the tower and pibal measurements is evident from a linear correlation of 0.95. In 59 of the 71 cases investigated at Green River, Utah, the tower recorded a wind speed lower than that obtained from the pibal observation. It is believed that these differences can be attributed to some combination of the following factors: space and time variability; tower shadow effect; balloon aerodynamics; terrain effects; systems error; and reduction process.

APPENDIX

A. Determination of Balloon Position

The position of a point in space can be determined by an azimuth and an elevation angle from one known location and an azimuth or elevation angle from another known location. However, since no tracking system is error-free, a more nearly correct solution can be obtained by increasing the amount of angular data. Statistical procedures are then applied to these data to obtain the most probable position.

The techniques used in this system determine the point, assumed to be the balloon position, such that the sum of the squares of distances from this point to each line of sight (as determined by the azimuth and elevation angle from the theodolite) is a minimum.

The position is calculated with respect to an earth-fixed Cartesian coordinate system, $OXYZ$, defined as

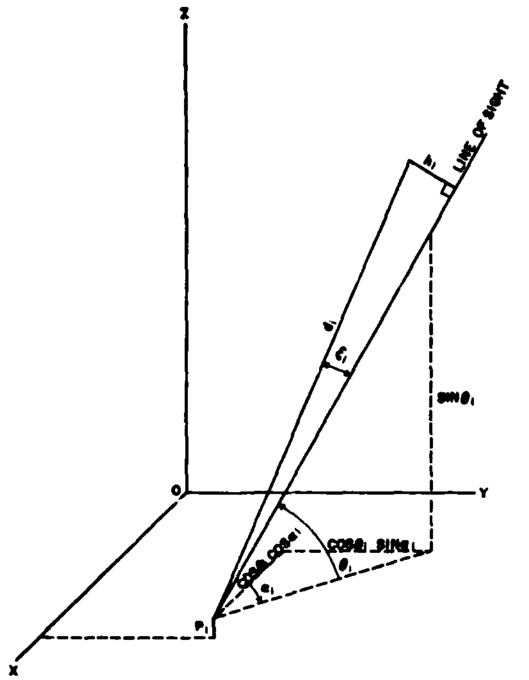

FIG. 6. Coordinate system utilized in the derivation of equations.

follows: Z lies along the zenith, X and Y are oriented such that $OXYZ$ is a right-hand system (see Fig. 6). In practice either X or Y is usually oriented positive northward; however, such a restriction is irrelevant to the solution.

The following notations are defined:

$P_i=(X_i,Y_i,Z_i)$ is the position of the ith theodolite.

(x,y,z) is the point to be determined.

α_i is the azimuth angle measured positive clockwise from a line through P_i parallel to the X axis to the projection on the X-Y plane of the line of sight from the ith theodolite.

θ_i is the elevation angle from the ith theodolite measured positive upward.

d_i is the distance from $(x,y,z,)$ to (X_i,Y_i,Z_i).

h_i is the distance from $(x,y,z,)$ to the line of sight from the ith theodolite.

ϵ_i is the angle between d_i and the ith line of sight.

ξ_i,η_i,ζ_i are the direction cosines of d_i.

a_i,b_i,c_i are the direction cosines of the ith line of sight.

From Fig. 6 it is clear that

$$\xi_i=\frac{x-X_i}{d_i},\quad \eta_i=\frac{y-Y_i}{d_i},\quad \zeta_i=\frac{z-Z_i}{d_i}; \tag{1}$$

$$a_i=\cos\theta_i\cos\alpha_i,\quad b_i=\cos\theta_i\sin\alpha_i,\quad c_i=\sin\theta_i, \tag{2}$$

and

$$h_i=d_i\sin\epsilon_i. \tag{3}$$

The function to be minimized is

$$F=\sum_{i=1}^{n} h_i^2. \tag{4}$$

By using the formula for the distance between two points and the properties of direction cosines, we have, after simplification

$$F=\sum_{i=1}^{n}\{(x-X_i)^2+(y-Y_i)^2+(z-Z_i)^2 -[a_i(x-X_i)+b_i(y-Y_i)+c_i(z-Z_i)]^2\}. \tag{5}$$

A necessary condition for F to be a minimum is

$$\frac{\partial F}{\partial x}=\frac{\partial F}{\partial y}=\frac{\partial F}{\partial z}=0. \tag{6}$$

Performing the operations in Eq. 6, we have, after considerable simplification

$$\begin{aligned} A_{11}x+A_{12}y+A_{13}z&=D_1,\\ A_{21}x+A_{22}y+A_{23}z&=D_2,\\ A_{31}x+A_{32}y+A_{33}z&=D_3, \end{aligned} \tag{7}$$

where

$$A_{11}=\sum_{i=1}^{n}(1-a_i^2)$$

$$A_{22}=\sum_{i=1}^{n}(1-b_i^2)$$

$$A_{33}=\sum_{i=1}^{n}(1-c_i^2)$$

$$A_{12}=A_{21}=-\sum_{i=1}^{n}a_ib_i$$

$$A_{13}=A_{31}=-\sum_{i=1}^{n}a_ic_i$$

$$A_{23}=A_{32}=-\sum_{i=1}^{n}b_ic_i$$

$$D_1=\Sigma[(1-a_i^2)X_i-a_ib_iY_i-a_ic_iZ_i]$$

$$D_2=\Sigma[(-a_ib_iX_i+(1-b_i^2)Y_i-b_ic_iZ_i]$$

$$D_3=\Sigma[-a_ic_iX_i-b_ic_iY_i+(1-c_i^2)Z_i].$$

The solution of the system of Eqs. (7) is the desired point $(x,y,z,)$.

B. Calculation of the Wind Components

The wind profile is determined by numerical differentiation of the position profile; the differentiation is performed componentwise. In general, the particular

numerical differentiation technique one uses depends on several factors which are peculiar to his particular problem. After consideration of data available, the accuracy obtainable, and the amount of computation required, we have chosen the following procedure.

Let h be an altitude at which the wind calculation is desired. Let (x_0,y_0,z_0) be the point on the position profile such that z_0 is nearest to h. Eleven successive (timewise) position points are chosen with (x_0,y_0,z_0) as their temporal midpoint. For notation purposes these points are given subscripts $(-5, -4, \ldots 0, \ldots, 5)$ where increasing subscript indicates increasing time. A parabola is fitted to these points, componentwise, and is differentiated to obtain the balloon speed, the negative of which is assumed to be the wind speed (the direction from which wind is blowing).

The mathematics involved is a straightforward application of least-squares techniques. Let T_0 be the time associated with the midpoint, and

$$t=\frac{T-T_0}{\Delta t}.$$

It suffices to find the coefficients of

$$x=a_0+a_1t+a_2t^2. \tag{8}$$

This is considerably simplified since it is required to evaluate only the derivative at $T=T_0$, i.e.,

$$\dot{x}=\frac{d}{dT}(a_0+a_1t+a_2t^2)=\frac{a_1}{\Delta t}+2\frac{a_2}{\Delta t}t, \tag{9}$$

and

$$\dot{x}|T_0=\frac{a_1}{\Delta t}. \tag{10}$$

One of the equations, readily obtainable by classical least-squares techniques, is

$$\Sigma tx=a_0\Sigma t+a_1\Sigma t^2+a_2\Sigma t^3. \tag{11}$$

We observe that $\Sigma t=\Sigma t^3=0$. Hence,

$$a_1=\frac{\Sigma tx}{\Sigma t^2}. \tag{12}$$

Thus

$$\dot{x}=\frac{a_1}{\Delta t}=\frac{\Sigma tx}{\Delta t\Sigma t^2}=\frac{1}{110\Delta t}\sum_{i=-5}^{5} ix_i \tag{13}$$

is the x component of the balloon speed. Thus,

$$W_x=\frac{1}{110\Delta t}\sum_{i=-5}^{5} ix_i,$$

and

$$W_y=\frac{1}{110\Delta t}\sum_{i=-5}^{5} iy_i. \tag{14}$$

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